

MECHANICS OF FLUIDS AND TRANSPORT PROCESSES

M. Lesieur

Turbulence in Fluids

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Turbulence in fluids

Stochastic and numerical modelling

By

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